Syllabus

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Course Information

Title: Special Topics – Mathematical Methods for Mechanical Engineering
Course Description

The primary objective of this class is to give a grounding in some basic mathematical analysis methods that are most relevant to mechanical engineers. Topics include linear algebra and vector spaces, tensor calculus, ordinary and partial differential equations. This presentation will therefore show what vector calculus is and, at the same time, give you an idea of what it's used for. A deep-seated conviction of mathematics for this course is that in any case, some mathematics is best discussed in a context that is not exclusively mathematical. Students will thoroughly understand gradient, curl, and Laplacian operators. Connections will be developed between vector operations and approximate solutions to partial differential equations which underly much of engineering theory and practice.

Course Objectives

At the conclusion of this course you will be able to:

- Conduct vector operations by hand and using computing
- Understand divergence of vector functions
- Apply the divergence theorem
- Compute line integrals and understand when path-dependency arises
- Compute the curl of vector functions and understand its relationship to other vector properties
- Compute gradient of vector functions
- Apply the gradient
- Relate vector calculus relationships to common approximations used for solution of PDE's
- (stretch goal) understand einstein notation for vector and tensor operations

Course Prerequisites

This course has no formal prerequisites but as a graduate engineering class assumes that
students have a basic grounding in vector calculus and differential equations (e.g. undergraduate Calculus III and Differential Equations). We will also make occasional use of numerical computation software such as MATLAB or python/numpy.

**Meeting Times:**

Class will meet from 9:00 to 10:20 am Monday and Wednesday in room 318 in the Liberal Arts building.

**Office Hours:**
tbd or by appointment

I'll set office hours within the first week or two of the semester. I strongly encourage you to come by with any questions or concerns you have. This time is yours. I'll have my office door open and may be working on something (probably coursework) but PLEASE INTERRUPT ME during these times. This time is yours to do anything I can which helps you learn. If I'm doing something else during office hours, it is only to avoid twiddling my thumbs while waiting for someone to stop by. There is nothing too small or too irrelevant to talk about, it is far better to come in early with a small question to get a good foundation.

**Course Communication**

Email is the best way to contact me. I will attempt to respond within 24-48 hours, please include the course title or number in your email subject.

**Course Resources**

**Required Materials**

*div, grad, curl, and all that: an informal text on vector calculus by H. M Schey*

Class will largely follow this book, with some material omitted or rearranged, and occasional substitution of optimization, fluid dynamic, dynamical system, and solid mechanics examples for the electrostatics-focused approach adopted by the book. Each class will have readings. I expect you to come to class having completed them and to have questions prepared. We will discuss questions, complete proofs, and work examples during class but I will not be lecturing the entirety of the book's material.

You may also find Advanced Engineering Mathematics by Erwin Kreyszig helpful

**Electronic Resources**
Modern engineering practice draws heavily on electronic resources broadly available including technical papers, content from online courses, technical forums, blog posts, and wikipedia. Recognizing this, you are encouraged to make careful use of these resources. They can be valuable references but their quality and notation can vary. Ultimately you will be tasked with developing solutions to novel problems in your careers so it is critical that an understanding of the theory of a solution is developed rather than simply stringing together code snippets or math solutions from stackoverflow. There are no restrictions on the resources you makes use of, but resources must be clearly documented, including a way for me to access a resource, a description of what was obtained from each resource, and how it was used.

If you use code snippets or work which is not originally yours, the work should be accompanied by references and comments identifying the source of the snippet in a way I can access (url) and a 1-2 sentence explanation of what that work does and why you selected it. For example in a piece of software code:

```python
# uniformly sample initial position on earth. I knew that sampling randomly from # latitude and longitude would over-sample the poles and under-sample the tropics. I figured that the earth is close enough to spherical that if # sampled on a sphere and normalized it to the earth's size that that would # close enough, but couldn't think of how to do this sampling, I found the # solution on stackexchange and translated it from R code to matlab. # https://stats.stackexchange.com/q/7988
n_samples = 1000
z_sample = 2 * np.random.rand(n_samples, 1) - 1
theta_sample = 2 * np.pi * np.random.rand(n_samples, 1) - np.pi
x_sample = np.sin(theta_sample) * np.sqrt(1 - z_sample**2.0)
y_sample = np.cos(theta_sample) * np.sqrt(1 - z_sample**2.0)
R_earth = 6378137.0
x = x_sample * R_earth
y = y_sample * R_earth
z = z_sample * R_earth
```

Note that you are still responsible for your solution being correct and working. For example – before I put this example in I did some basic checks (plotting the output and checking the statistics against a uniform lat-lon sample to make sure they were different).

**Course Structure and Sequence**

The course will begin with a review of vector and tensor fundamentals, and a brief discussion of vector spaces. We will then discuss a number of topics in vector calculus. The course will
wrap up with application of vectors and vector calculus in PDE’s and a self-identified project which makes use of vector analysis.

Vectors, Tensors, and Vector Spaces

In the first part of the course we will cover required definition of vectors and tensors. We will briefly define vector spaces and review basic vector and tensor operations. We will also define what a vector function is.

Vector Calculus

Much of the class will be devoted to exploring a number of key ideas and relationships in vector calculus. This will include theorems of engineering significance including the divergence theorem and stokes’ theorem. We will explore the gradient and potential functions.

Underpinnings of Engineering Theory and Practice

Finally, we will wrap up with exploration of connections between vectors and vector calculus concepts and the theory and practice which underlies practical approaches to solving some partial differential equations (and thus underpin much of engineering). If we get to it we’ll also review a compact notation for tensor operations which is commonly used in continuum mechanics.

Assignments and Evaluation

Progress in this course will be evaluated through a sequence of exercises, two exams, and a final project.

Class Exercises

You will each lead the class during a portion of an in-class exercise. You will be expected to:

- Analyse what a problem is looking for, identifying possible approaches
- Sketch out the steps you expect to be part of the solution (without necessarily solving the problem)
- Identifying where you expect the greatest difficult to be (again, prior to solving the problem)

We will then take this as a template and attempt solution as a class.
Independent Exercises

A series of independent exercises will accompany the ones we work on as a class. These are opportunities to test your understanding and practice (if you are having difficulty I strongly recommend working extra problems from the book until you are comfortable with the exercises). Each exercise will have two due dates – on the first you will turn in your work, on the second you will complete a self-evaluation of your work after being provided a solution. Exercises will be scored on a two element basis:

- Did you make a good faith attempt to complete the exercise?
- Did you provide a meaningful evaluation of your work?

Your self-assessment should highlight anything that was incomplete or incorrect, as well as any misunderstanding that was corrected by reviewing the solution and evaluating your own work. I will review your self-assessment and add comments as necessary if you've missed something. Your exercise score will depend solely on a good faith effort at engaging with these objectives and not on your solutions themselves.

Exams

There will be two exams held during a class period. You have two options for resources available during the exam. You may bring as many notes that you have personally prepared into the exam as you like and use them during the exam, turning them in as part of your solution. Alternatively, you may use any resources you like (including any on the internet) but must clearly document any reference which contributes to your solution (more details on this as we near an exam). You must select which option you will use prior to the start of the exam.

Project

You will also complete a self-identified project at the end of the semester employing vector analysis. I encourage you to draw this project from personal interests or your graduate research topics. This project will have a number of milestones leading up to a final report due at the conclusion of the semester.

Grade Expectations

Rubrics for evaluation of course assignments will be constructed so that a “C” level grade indicates a satisfactory solution to the assigned problem but without demonstrating an understanding of the theoretical background for the problem and its solution. A “B” level
grade will indicate both solution of the assigned problem and an understanding of its theoretical properties. “A” level grades will indicate that in addition to “C” and “B” level mastery that you understand the limitations of the solution developed.

Grade Assignment

Nominal Weights

<table>
<thead>
<tr>
<th>Deliverable</th>
<th>Weight</th>
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<tbody>
<tr>
<td>Class Exercises</td>
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<td>Independent Exercises</td>
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<td>Exams</td>
<td>0.4</td>
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<tr>
<td>Project</td>
<td>0.25</td>
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Preliminary Schedule

<table>
<thead>
<tr>
<th>Day</th>
<th>Exercise</th>
<th>Topic</th>
<th>Pre-Reading</th>
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<tbody>
<tr>
<td>2023-08-28</td>
<td></td>
<td>Vectors and Tensors</td>
<td>introduction</td>
</tr>
<tr>
<td>2023-08-30</td>
<td></td>
<td>Vector Operations</td>
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</tr>
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<td>No class</td>
<td></td>
</tr>
<tr>
<td>2023-09-06</td>
<td></td>
<td>Vector Spaces</td>
<td></td>
</tr>
<tr>
<td>2023-09-11</td>
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<td>Vector Functions</td>
<td>vector functions</td>
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<tr>
<td>2023-09-13</td>
<td></td>
<td>Normal Vectors</td>
<td>the unit normal vector</td>
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<tr>
<td>2023-09-18</td>
<td>2 due</td>
<td>Surface Integrals</td>
<td>definition of surface integrals and evaluating surface integrals</td>
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<tr>
<td>2023-09-20</td>
<td>2 feedback</td>
<td>Gauss’ Law</td>
<td>gauss’ law, flux, using Gauss’ law?</td>
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<tr>
<td>Day</td>
<td>Exercise</td>
<td>Topic</td>
<td>Pre-Reading</td>
</tr>
<tr>
<td>---------------</td>
<td>----------</td>
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<tr>
<td>2023-09-25</td>
<td></td>
<td>Divergence</td>
<td>the divergence through the del notation</td>
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<tr>
<td>2023-09-27</td>
<td></td>
<td>The Divergence theorem</td>
<td>the divergence theorem</td>
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<tr>
<td>2023-10-02</td>
<td>3 due</td>
<td>Line Integrals</td>
<td>work and line integrals, line integrals involving vector functions</td>
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<tr>
<td>2023-10-04</td>
<td>3 feedback</td>
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<td>path independence and the curl</td>
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<td>2023-10-09</td>
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<td>Exam 1 (through the divergence theorem)</td>
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<td>2023-10-11</td>
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<td>Circulation</td>
<td>the curl in? through differential form of?</td>
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<td>2023-10-16</td>
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<td>Stokes’ theorem</td>
<td>stokes’ theorem and an application of?</td>
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<tr>
<td>2023-10-18</td>
<td>4 due</td>
<td>Manifolds and Connection</td>
<td>Stokes’ theorem and simply connected? and path independence and?</td>
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<td>2023-10-23</td>
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<td>gradients</td>
<td>line integrals and the gradient</td>
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<td>2023-10-25</td>
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<td>laplacian</td>
<td>finding?”and using laplace’s equation</td>
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<tr>
<td>2023-10-30</td>
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<td>interpreting the gradient</td>
<td>directional derivatives and the gradient</td>
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<td>2023-11-01</td>
<td></td>
<td>interpreting the gradient</td>
<td>geometric significance of the gradient</td>
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</tbody>
</table>
### Course Policies

#### Technology Requirements

Some exercises may benefit from use of computer numerical systems. I may also provide some examples illustrating class concepts as MATLAB code. Your final project may require you to implement the concepts from this class in software, depending on what you choose.

If you do not have access to a laptop, you can borrow one from [the library at this link](https://birdjj.github.io/courses-math_for_mech_engr/syl...).

Your project report should be submitted as zip files containing a PDF report and any computer code you've written through the blackboard system.

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<table>
<thead>
<tr>
<th>Day</th>
<th>Exercise</th>
<th>Topic</th>
<th>Pre-Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>2023-11-06</td>
<td>5 due</td>
<td>Gradients and Taylor expansion</td>
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</tr>
<tr>
<td>2023-11-08</td>
<td>5 feedback</td>
<td>Solving PDE's with Taylor expansion</td>
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<tr>
<td>2023-11-13</td>
<td></td>
<td>Exam 2 (through interpreting gradients)</td>
<td></td>
</tr>
<tr>
<td>2023-11-15</td>
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<td>Modal expansion</td>
<td>TBD</td>
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<tr>
<td>2023-11-20</td>
<td></td>
<td>Modal expansion and matrices</td>
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<tr>
<td>2023-11-22</td>
<td>6 due</td>
<td>Null spaces and eigenvalues</td>
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<tr>
<td>2023-11-27</td>
<td>6 feedback</td>
<td>Einstein Notation</td>
<td>TBD</td>
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<tr>
<td>2023-11-29</td>
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<td>reserved</td>
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</tr>
<tr>
<td>2023-12-04</td>
<td></td>
<td>reserved</td>
<td></td>
</tr>
<tr>
<td>2023-12-06</td>
<td></td>
<td>reserved</td>
<td></td>
</tr>
</tbody>
</table>
Course Attendance Policy

There will be no formal attendance taken, but you are expected to be engaged in the class discussions.

Late Work Policy

Assignments may be accepted at my discretion provided that you contact me more than 24 hours in advance and receive an extension. Absent an emergency or prior extension, work that is simply not turned in on time will not be accepted after the deadline. If you have an emergency get in touch as soon as practical and we’ll work something out.

Accommodations Policy

The University is committed to providing reasonable accommodations and auxiliary services to students, staff, faculty, job applicants, applicants for admissions, and other beneficiaries of University programs, services and activities with documented disabilities in order to provide them with equal opportunities to participate in programs, services, and activities in compliance with sections 503 and 504 of the Rehabilitation Act of 1973, as amended, and the Americans with Disabilities Act (ADA) of 1990 and the Americans with Disabilities Act Amendments Act (ADAAA) of 2008. Reasonable accommodations will be made unless it is determined that doing so would cause undue hardship on the University. Students requesting an accommodation based on a disability must register with the UTEP Center for Accommodations and Support Services (CASS). Contact the Center for Accommodations and Support Services at 915-747-5148, or email them at cass@utep.edu, or apply for accommodations online via the CASS portal.

COVID 19 Precautions

You are expected to adhere to university guidance on COVID 19 precautions available at: https://www.utep.edu/resuming-campus-operations/ Guidance and policy with respect to COVID may change throughout the semester.

If you have tested positive for COVID 19 or have reason to suspect you may have COVID 19 (e.g. because of symptoms or close contact with an individual who has COVID 19) you are expected to stay home as directed by https://www.cdc.gov/coronavirus/2019-ncov/your-health/quarantine-isolation.html. Contact me and we will arrange appropriate accommodations.

Students who are considered high risk according to CDC guidelines and/or those who live...
with individuals who are considered high risk may contact Center for Accommodations and Support Services (CASS) to discuss temporary accommodations for on-campus courses and activities.

Academic Integrity

Academic dishonesty is prohibited and is considered a violation of the UTEP Handbook of Operating Procedures. It includes, but is not limited to, cheating, plagiarism, and collusion. Cheating may involve copying from or providing information to another student, possessing unauthorized materials during a test, or falsifying research data on laboratory reports. Plagiarism occurs when someone intentionally or knowingly represents the words or ideas of another as one's own. Collusion involves collaborating with another person to commit any academically dishonest act. Any act of academic dishonesty attempted by a UTEP student is unacceptable and will not be tolerated. All suspected violations of academic integrity at The University of Texas at El Paso must be reported to the Office of Student Conduct and Conflict Resolution (OSCCR) for possible disciplinary action. To learn more, please visit HOOP: Student Conduct and Discipline.