

**University of Texas at El Paso**  
**College of Education**  
**Department of Teacher Education**  
**MTED 5326 (23321) Cultural-Historical Epistemology and Didactics of Mathematics**  
**Spring 2019**

"It is not knowledge, but the act of learning. Not possession, but the act of getting there  
that grants the greatest enjoyment." - *Karl Friedrich Gauss, Mathematician*

**Class meeting time:** M 5:30 p.m. – 8:20 p.m.

**Class meeting place:** College of Education, EDUC 402

**Professor:** Dr. David J. Carrejo, Ph.D.

**Office:** College of Education, EDUC 802

**Phone:** 747-5856

**E-mail:** [dcarrejo@utep.edu](mailto:dcarrejo@utep.edu) (best means of contact)

**Office Hours:** M 3:30 p.m. – 5:00 p.m., R 9:30 a.m. – 11:00 a.m., or by appointment

**This syllabus is subject to change as needed. Any changes to the syllabus will be announced in class.**

No cellular phones or beepers are permitted in class.

If you have or suspect a disability and need accommodations you should contact the Center for Accommodations and Support Services at 747- 5148 or at [cass@utep.edu](mailto:cass@utep.edu) or come by Union East Building, Room 106.

### **Required Texts**

- Required readings will be available through open-source websites, through Blackboard, and through electronic journals accessible through the UTEP library website.
- *Geogebra: Dynamic Software for Everyone [Classic 5.0]*. Available: [www.geogebra.org](http://www.geogebra.org) This is free, open-source software for both Mac and Windows platforms.
- The majority of course materials (handouts, Geogebra files, and web links) will be posted on Blackboard.

### **Course Description**

The purpose of this course is for students to encounter a wide variety of historical situations and contexts which have given rise to mathematical concepts that now form the basis of our required high school mathematics curriculum (grades 9 - 12). Because this course will focus on the historical origins of our modern high school curricula, there will be a special emphasis on the mathematical and scientific developments that occurred in seventeenth century Europe, the period of the "Scientific Revolution." As we shall see, the origins of most of our current mathematics curriculum (up to and including calculus), are to be found there.

Students will be given readings and a variety of projects which will allow them to investigate original source materials and draw their own conclusions about the direction of this historical genesis. Herein lies our discussions of "genetic epistemology." It is my hope that such historical analysis will help students rethink (critically) the current high school mathematics that is central in our current culture. However, just as importantly, we will discuss the implications that such historical analysis has on our rethinking the EC-8 mathematics curriculum. Herein lies our discussion of "didactics" (or the teaching) of mathematics.

### **A Guiding Philosophic Principle**

Mathematics is a dialogue between grounded activity and systematic inquiry. Grounded activities are situations arising from physical activity with strings, sticks, architecture, machines, dice, finance, weaponry, communications and anything else under the sun. Systematic inquiry involves the fullest possible use of the tools that are commonly available in a culture, including both physical tools (such as rulers, scales, clocks, thermometers, projectors, calculators, and computers) and linguistic tools (such as words, numbers, symbols, drawings, diagrams, tables, and graphs). This dialogue is a deep and essential element of what it means to be human, and all humans are engaged in some form of this dialogue. This is what makes mathematics a human endeavor. The refinement of the expression of this dialogue is achieved through a broader set of physical experiences, and clearer communication tested in the context of social interaction. Thus a broad view of the historical genesis of mathematics will both clarify and redefine what mathematics means to us.

### **Course Goals**

- 1) To increase student understanding of epistemology and its role in the evolution and development of mathematical knowledge.
- 2) To increase student understanding of research and scholarly work in the history of mathematics.

- 3) To allow students to engage in class discussions and assignments which require an examination of both mathematical content knowledge and pedagogical content knowledge.

### Course Objectives

Upon completion of this course, students will be able to:

- 1) Examine and re-design their pedagogical approaches from an epistemological perspective.
- 2) Learn about important technologies related to pedagogy in mathematics.

### Policy on Academic Dishonesty

The University of Texas at El Paso prides itself on its standards of academic excellence. In all matters of intellectual pursuit, UTEP faculty and students must strive to achieve based on the quality of work produced by their individual. In the classroom and in all other academic activities, students are expected to uphold the highest standards of academic integrity. Any form of scholastic dishonesty is an affront to the pursuit of knowledge and jeopardizes the quality of the degree awarded to all graduates of UTEP. It is imperative, therefore, that all faculty, insist on adherence to these standards.

Any student who commits an act of scholastic dishonesty is subject to discipline. Scholastic dishonesty includes, but is not limited to cheating, plagiarism, collusion, the submission for credit of any work or materials that are not attributable in whole or in part to another person, taking an examination for another person, any act designed to give unfair advantage to a student or the attempt to commit such acts. Proven violations of the detailed regulations, as printed in the Handbook of Operating Procedures (HOP) and available in the Office of the Dean of Students, may result in sanctions ranging from disciplinary probation, to failing grades on the work in question, to failing grades in the course, to suspension or dismissal among others.

### Course Requirements

- It is expected that students will attend **all classes** and actively participate in working on projects and class discussions. Students are expected to prepare for each class session. **Lateness to class is strongly discouraged. With the emphasis on collegiality it is important that all group members be in class to contribute to the group's effort in developing an understanding of what it means to teach mathematics effectively.**
- Assignments are due on the specified dates. **Late assignments will not be accepted.**
- The schedule of topics and reading assignments may change over the course of the semester. **Any changes to the syllabus will be announced in class. Every student is responsible for these changes whether or not the student is present in class.**
- **Type or word-process all written assignments.** All research/writing assignments should be double spaced with a 12 point font. Number your pages, preferably using a header or footer. Correct grammar and spelling are expected. Further guidelines for the final project will be provided in class.
- **Homework should be turned in as hard copy.** Given the nature of the mathematics problems, the traditional paper and pencil method of completing the assignments is preferred, including any graphs, charts, or tables. Geometric constructions should also be completed by paper and pencil unless otherwise specified (e.g. using Geogebra).

### Attendance Policy

Regular attendance and your active participation are critical for the success of a course of this type, and you cannot miss more than 2 class sessions. Your overall course grade will be lowered by one full letter grade for each absence beyond 2 absences. In other words, if you earned an "A" as your course grade but missed 3 classes, your course grade would be a "B." If you MUST miss a class, please contact me (in advance, if possible) by phone and e-mail. Exceptions will be made only for medical emergencies and compassionate reasons (cases will vary). Your participation grade is dependent on your attendance. Please note that "Incomplete" is **RARELY** granted.

### Course Assignments

#### *1. Reflection papers/Blackboard postings*

Over the course of the semester, you will be asked to synthesize what you are reading and integrate it with discussions held in class. At several points throughout the semester (see calendar), you will be asked to prepare a short (1-2 page) written reflection on an essay question that I will pose in class or post on Blackboard. The question will focus on a topic or topics discussed in class.

## 2. Homework (problem-based activities)

During this course you will work on historically-based mathematics activities provided by me. You will be assigned homework assignments based on these activities. The homework assignments and due dates will be announced in class; typically, the homework will be due the following class day. All homework assignments will be posted on Blackboard. Given the nature of the assignments, I prefer that you turn in your homework assignments in hard-copy format. In some cases, electronic format will be acceptable.

## 3. Final Project

The focus of the final project is to choose a specific mathematics topic from an historical perspective, refined by an epistemological and didactic perspective, and design a sequence of lessons based on your choice of topic. You want to focus on how your topic is relevant to mathematics education research, given that we have examined the didactics and epistemology relevant to our current mathematics curriculum as well as a specific, highly relevant era of the history of mathematics.

You will be responsible for writing a **minimum 10 page** paper supporting your choice of topic from a history of mathematics (i.e. a paper that justifies why you believe the topic you've chosen is important and how it is based on a sound understanding of the history of 17<sup>th</sup> century mathematics, its epistemology and didactics). You are free to choose any era or time period within the history of mathematics. However, be sure that your choice of topic can be aligned with our current studies of the current mathematics curriculum.

For your paper, you must include an introduction section that introduces the content and the significance of the topic for mathematics education. Include a theoretical framework that focuses on student/teacher learning for your given topic (i.e. why it is important for students and teachers to learn this topic or topics and why they should learn it based on your understanding of 17<sup>th</sup> century mathematics and the historical context that we analyze). You are required to have **a minimum of five solid references** from peer-reviewed journals or edited books.

I require that you construct a sequence of activities (**a minimum of two**) based on your research. Topics will receive final approval from me. The completed sequence must involve either a technology (computer) component and/or two "hands-on" components (which can involve technology).

**Further details about the project will be given in class along with continuing guidance from me.** You will submit your project **on Monday, May 13<sup>th</sup>**, the final exam day. All materials related to the final project **must be** submitted on that day.

## Grades

In this course all grades are important, but some assignments take more time and thought so therefore some may have a different weight.

- **Attendance** 20%
- **Reflections** 10%
- **Homework Assignments** 30%
- **Final Project** 40%

<b>Grade Distribution:</b>	<b>Grade</b>	<b>Percentage (%)</b>
	<b>A</b>	<b>90 - 100</b>
	<b>B</b>	<b>80 - 89.9</b>
	<b>C</b>	<b>70 - 79.9</b>
	<b>D</b>	<b>60 - 69.9</b>
	<b>F</b>	<b>below 60</b>

## General calendar

Topics, assigned readings, and due dates *are subject to change*. All assignments are **DUE ON THE DATE INDICATED**.

**MI** = Mathematical Intentions website ([www.quadrivium.info](http://www.quadrivium.info)) **R** = Reading from bibliography list

DATE	TOPIC	DISCUSSION FOCUS	ASSIGNMENTS (READINGS AND HOMEWORK)
<b>January 28</b>	Introductions Overview of the course	<ul style="list-style-type: none"> <li>Our current secondary mathematics curriculum</li> <li>Didactics, epistemology, and Ethnomathematics defined</li> </ul>	
<b>February 4</b>	History of mathematics, didactics of mathematics, and their role(s) in STEM education	<ul style="list-style-type: none"> <li>STEM education research goals and agendas</li> </ul>	<b>R:</b> Dennis (2000) <b>R:</b> Ernest (2002)
<b>February 11</b>	History of mathematics History of mathematics education	<ul style="list-style-type: none"> <li>The printed word</li> <li>The Scientific Revolution</li> <li>Schooling in North America</li> </ul>	<b>MI:</b> Lecture 2, Lecture 3 <b>R:</b> Ellerton & Clements (2012)
<b>February 18</b>	Geometry in the 17 <sup>th</sup> century	<ul style="list-style-type: none"> <li>The emergence of measurement and “Observable units”</li> <li>Geometric constructions</li> <li>GeoGebra technology</li> </ul>	<b>MI:</b> Lecture 4, Lecture 5 <b>MI:</b> Geometric constructions <b>MI:</b> Similarity, geometric arithmetic, and the geometric mean <b>Reflection 1 due</b>
<b>February 25</b>	Geometry in the 17 <sup>th</sup> century Galileo Galilei	<ul style="list-style-type: none"> <li>Square roots</li> <li>Logarithms</li> <li>Mathematics and modeling nature</li> <li>Experiencing motion</li> <li>GeoGebra technology</li> </ul>	<b>MI:</b> Square root calculations <b>MI:</b> Slide rules and logarithm tables <b>MI:</b> Descartes’s logarithm machine <b>Homework 1 due</b>
<b>March 4</b>	Rene Descartes and <i>La Geometrie</i>	<ul style="list-style-type: none"> <li>Mathematics and modeling nature</li> <li>Functions of a curve</li> </ul>	<b>R:</b> Dennis, D. (1997)
<b>March 11</b>	Rene Descartes and the conic sections	<ul style="list-style-type: none"> <li>Mathematics and modeling nature</li> <li>History of the conic sections</li> </ul>	<b>MI:</b> Apollonius <b>MI:</b> Parabolas and coordinates <b>Homework 2 due</b>
<b>March 18</b>	<b>SPRING BREAK – NO CLASS</b>		
<b>March 25</b>	Rene Descartes and the conic sections Blaise Pascal’s geometry	<ul style="list-style-type: none"> <li>The arithmetical triangle</li> <li>Algebra</li> </ul>	<b>MI:</b> Ellipses <b>MI:</b> Alhazen’s summation formulas <b>R:</b> Pengelley (2009) <b>Reflection 2 due</b>
<b>April 1</b>	Blaise Pascal’s geometry	<ul style="list-style-type: none"> <li>The arithmetical triangle</li> <li>Al-Hazen summation</li> </ul>	<b>R:</b> Dennis, D. & Confrey, J. (1996) <b>MI:</b> Negative exponents <b>Homework 3 due</b>
<b>April 8</b>	John Wallis and Isaac Newton	<ul style="list-style-type: none"> <li>Mathematics and modeling nature</li> <li>Characteristic Ratios and Tables</li> </ul>	<b>R:</b> Dennis, D. & Confrey, J. (1996) <b>MI:</b> Negative exponents
<b>April 15</b>	Isaac Newton	<ul style="list-style-type: none"> <li>Mathematics and modeling nature</li> <li>Geometry and algebra: The calculus</li> </ul>	<b>MI:</b> Newton’s binomial series <b>Reflection 3 due</b>
<b>April 22</b>	Isaac Newton	<ul style="list-style-type: none"> <li>Mathematics as language</li> <li>Geometry versus algebra</li> </ul>	<b>MI:</b> Newton and empirical interpolation
<b>April 29</b>	Gottfried W. Leibnitz	<ul style="list-style-type: none"> <li>The Cycloid</li> <li>Trigonometry &amp; Transmutation</li> </ul>	<b>MI:</b> The Cycloid <b>MI:</b> Sines, Circles, and Transmutations <b>Homework 4 due</b>
<b>May 6</b>	What was worth knowing by the 18 <sup>th</sup> century (and beyond)?	<ul style="list-style-type: none"> <li>Didactics and designing mathematical activity</li> </ul>	<b>MI:</b> Lecture 45 <b>R:</b> Winslow, C. (2007) <b>R:</b> Bartolini Bussi & Bazzini (2003) <b>Homework 5 due</b>
<b>May 13</b>	Exam day		<b>Final Projects Due</b>

## List of Required Readings

### February 4

Dennis, D. (2000). The role of historical studies in mathematics and science educational research. In Kelly & Lesh, R. (Eds.), *Research Design in Mathematics and Science Education* (pp. 799-813). Lawrence Erlbaum Associates.  
**Download this article:** <http://www.quadrivium.info/MathInt/MathIntentions.html>

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal*, 15.  
**Download this article:** <http://people.exeter.ac.uk/PErnest/pome15/contents.htm>

### February 11

Ellerton, N., & Clements, M.A. (2012). Translating the cyphering book tradition to North America [book chapter]. In *Rewriting the History of School Mathematics in North America 1607 – 1861*. Springer. **Available: Blackboard**

### March 4

Dennis, D. (1997) Rene Descartes' curve-drawing devices: Experiments in the relations between mechanical motion and symbolic language. **Available:** <http://www.quadrivium.info/MathHistoryIndex.html>

### March 25

Pengelley, D. (2009). Pascal's treatise on the arithmetical triangle: Mathematical induction, combinations, the Binomial Theorem and Fermat's Theorem. *MAA Notes* (no. 74). 185 – 196. **Available:** [http://www.math.nmsu.edu/hist\\_projects/pascalII.pdf](http://www.math.nmsu.edu/hist_projects/pascalII.pdf)

### April 1 and 8

Dennis, D., & Confrey, J. (1996). The creation of continuous exponents: A study of the methods and epistemology of John Wallis. **Available:** <http://www.quadrivium.info/mathhistory/Wallis.pdf>

### May 6

Winslow, C. (2007). Didactics of mathematics: An epistemological approach to mathematics education. *The Curriculum Journal*, 18(4), 523 – 536. **Available: UTEP library online e-journal database**

Bartolini Bussi, M.G., & Bazzini, L. (2003). Research, practice, and theory in didactics of mathematics: Towards dialogue between different fields. *Educational Studies in Mathematics*, 54, 203 – 223. **Available: UTEP library online e-journal database**